Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

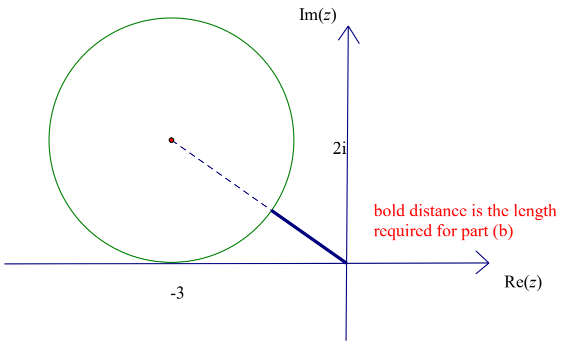
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(5x-3)^2 < 2^2(3x-7)^2$, or corresponding quadratic equation, or pair of linear equations $(5x-3) = \pm 2(3x-7)$	**B1**	$11x^2 - 138x + 187 > 0$.
	Solve a 3-term quadratic, or solve **two** linear equations for x	**M1**	If no working is shown, the M1 is implied by the correct roots for an incorrect quadratic.
	Obtain critical values $x = \frac{17}{11}$ and $x = 11$	**A1**	Accept 1.55 or better.
	State **final** answer $x < \frac{17}{11}$, $x > 11$	**A1**	Strict inequality required. In set notation, allow notation for open sets but not for closed sets e.g. accept $\left(-\infty, \frac{17}{11}\right) \cup (11, \infty)$ or $(-\infty, \frac{17}{11}[\cup]11, \infty)$ but not $(-\infty, \frac{17}{11}] \cup [11, \infty)$. Allow 'or' but not 'and'. Accept $\cup$. Final A0 for $\frac{17}{11} > x > 11$. Exact values expected but ISW if exact inequalities seen followed by decimal approx.
	Alternative Method for Question 1		
	Obtain critical value $x = 11$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x = \frac{17}{11}$ similarly	**B2**	Accept decimal value.
	State final answer $x < \frac{17}{11}$, $x > 11$	**B1**	Strict inequality required. See notes above.
		4	

Question	Answer	Marks	Guidance
2	Use law of the logarithm of a power, quotient or product	M1	Must be used correctly on a **correct** term. e.g. M1 for $2\ln x = \ln x^2$ but M0 for $2\ln x - \ln 2 = 2\ln\frac{x}{2}$. M0 for $\ln(2x^2-3) = \ln 2x^2 - \ln 3$ $= \ln 2 + 2\ln x - \ln 3$.
	Remove logarithms and obtain a correct equation in x	A1	e.g. $2x^2 - 3 = \frac{x^2}{2}$.
	Obtain final answer $x = \sqrt{2}$ only	A1	If $x = -\sqrt{2}$ is mentioned, it must be rejected.
		3	

Question	Answer	Marks	Guidance
3(a)		B1	Show a circle with centre $-3 + 2\text{i}$. Allow for a curved figure with 'centre' in roughly the correct position. Accept marks or numbers on axes, coordinates of centre shown. B0B1 available for axes the wrong way round (and M1 A1 in part **(b)**).
	Show a circle with radius 2	B1 FT	FT centre not at the origin. Allow 'near miss' on x axis. Different scales on axes require an ellipse for B1 B1. Scales on the axes and any label of the radius must be consistent for B1 B1. Correct circle shaded scores B1 B0.
		2	

Question	Answer	Marks	Guidance
3(b)	Carry out a correct method for finding the least value of $\|z\|$	**M1**	e.g. distance of centre from origin – radius or find point of intersection of circle and $3y = -2x$ and use Pythagoras. If they subtract the wrong way round M0. If their diagram is a reflection or a rotation of the correct diagram, M1 A1 is available (requires equivalent work). Any other circle M0.
	Obtain answer $\sqrt{13}-2$ or $\sqrt{17-4\sqrt{13}}$	**A1**	Or exact equivalent e.g. $\sqrt{17-\frac{26}{3}\sqrt{\frac{36}{13}}}$. Correct solution only. Allow A1 if exact answer seen and then decimal given.
		2	

Question	Answer	Marks	Guidance
4	Use correct double angle formula to obtain an equation in $\cos\left(\frac{x}{2}\right)$ only	*M1	e.g. $2\left(2\cos^2\left(\frac{x}{2}\right)-1\right)-\cos\left(\frac{x}{2}\right)=1$.
	Obtain a 3 term quadratic in $\cos\left(\frac{x}{2}\right)$,	A1	e.g. $4\cos^2\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)-3=0$. Allow $4\cos^2 u-\cos u-3=0$. Condone $\frac{x}{2}=x$.
	Obtain $\cos\left(\frac{x}{2}\right)=-\frac{3}{4}$ and $\cos\left(\frac{x}{2}\right)=1$	A1	Allow answer in u e.g. $(4\cos u+3)(\cos u-1)$ and condone $\frac{x}{2}=x$.
	Solve for the **original** x	DM1	Must see evidence of doubling, not halving.
	Obtain $x = 0$ and 4.84 and no others in the interval	A1	Ignore any answers outside interval. Accept AWRT 4.84. Accept 1.54π. Must be in radians. 277.2 indicates M1 but is A0.
	Alternative Method for Question 4		
	Use correct double angle formula to obtain an equation in $\cos x$ only	*M1	e.g. $2\cos x-1=\sqrt{\frac{\cos x+1}{2}}$.
	Obtain a 3 term quadratic in $\cos x$,	A1	e.g. $8\cos^2 x-9\cos x+1=0$.
	Obtain $\cos x=\frac{1}{8}$ and $\cos x = 1$	A1	
	Solve for x	DM1	
	Obtain answers $x = 0$ and 4.84 and no others in the interval	A1	Ignore any answers outside interval. Accept AWRT 4.84. Must be in radians. 277.2 is A0.
		5	

Question	Answer	Marks	Guidance
5(a)	Substitute $2 + y\text{i}$ in $a^3 - a^2 - 2a$ and attempt expansions of a^2 and a^3	M1	$a^2 = 4 + 4y\text{i} - y^2$ $a^3 = 8 + 12y\text{i} - 6y^2 - y^3\text{i}$. If using $a\left(a^2 - a - 2\right)$ must then expand fully. Must see working.
	Use $\text{i}^2 = -1$	M1	Seen at least once (e.g. in squaring).
	Obtain **final** answer $-5y^2 + (6y - y^3)\text{i}$	A1	Or simplified equivalent e.g. $6y\text{i} - 5y^2 - y^3\text{i}$. Do not ISW.
		3	No evidence of working for the square or the cube can score SC B1 for the correct answer.
5(b)	Equate *their* $-5y^2$ to -20 and solve for y	M1	Need to obtain a value for y. Available even if *their* y is not real.
	Obtain $y = -2$	A1	From correct work. Allow after incorrect f(a) if the real part was correct. Condone ± 2 with positive not rejected.
	Obtain **final** answer $\arg a = -\dfrac{\pi}{4}$	A1	Correct only (must have rejected y positive). OE e.g. $-\frac{\pi}{4} \pm 2n\pi$. Accept -0.785, 5.50. Allow after incorrect f(a) if the real part was correct. Accept degrees. Do not ISW.
		3	

Question	Answer	Marks	Guidance
6(a)	Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 1$	M1	Need to evaluate at both points, but M1 still available if one value incorrect. Use of degrees is M0. Correct use of a smaller interval is M1. If using $g(x) - f(x)$, there needs to be a clear indication of the comparison being made e.g. by listing values in a table. Embedded values 0.5 and 1 are not sufficient. 3.92 and 1.83 alone are not sufficient.
	Complete the argument correctly with conclusion about change of sign or change of inequalities and with correct calculated values. Can all be in symbols – an explanation in words is not required.	A1	e.g. $3.92 > 1.5$, $1.83 < 3$ or $2.42 > 0$, $-1.17 < 0$.
		2	
6(b)	State $x = \frac{1}{3}\left(x + 4\tan^{-1}\frac{1}{3x}\right)$	M1	Or rearrange $\cot\left(\frac{x}{2}\right) = 3x$ as far as $2x = 4\tan^{-1}\left(\frac{1}{3x}\right)$
	Rearrange to the given equation $\cot\left(\frac{x}{2}\right) = 3x$ Need intermediate step between $\frac{x}{2} = \tan^{-1}\frac{1}{3x}$ and $\cot\left(\frac{x}{2}\right) = 3x$	A1	Or continue rearrangement to $x = \frac{1}{3}\left(x + 4\tan^{-1}\frac{1}{3x}\right)$ and state iterative formula of $x_{n+1} = \frac{1}{3}\left(x_n + 4\tan^{-1}\frac{1}{3x_n}\right)$ **AG**
		2	

Question	Answer	Marks	Guidance
6(c)	Use the iterative process correctly at least once	M1	Obtain one value and substitute that back in to obtain a second value. Working in degrees is M0.
	Obtain final answer 0.79	A1	Must be to 2 d.p.
	Show sufficient iterations to at least 4 d.p. to justify 0.79 to 2 d.p. or show there is a sign change in the interval (0.785, 0.795)	A1	e.g. 1, 0.7623, 0.8037, 0.7921, 0.7951, 0.7943, 0.7945 or 0.5, 0.9506, 0.7665, 0.8024, 0.7924, 0.7950, 0.7944, 0.7945 or 0.75, 0.8076, 0.7911, 0.7954, 0.7943, 0.7946, 0.7945 . Condone truncation. Allow recovery. Condone minor differences in the final d.p.
		3	If they do the iteration in **(b)** but restate the conclusion here, no marks in **(b)** but could score 3/3 for **(c)**.

Question	Answer	Marks	Guidance
7(a)	State or imply $6y\frac{\mathrm{d}y}{\mathrm{d}x}$ as the derivative of $3y^2$	**B1**	Allow y' for $\frac{\mathrm{d}y}{\mathrm{d}x}$ throughout. Accept $\frac{\partial f}{\partial x}=6x+4y$.
	State or imply $4x\frac{\mathrm{d}y}{\mathrm{d}x}+4y$ as the derivative of $4xy$	**B1**	Accept $\frac{\partial f}{\partial y}=4x+6y$.
	Equate derivative of LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	Allow an extra $\frac{\mathrm{d}y}{\mathrm{d}x}$ in front of their differentiated equation. Allow if '= 0' is implied but not seen. Allow $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{\partial f}{\partial x}\Big/\frac{\partial f}{\partial y}$
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3x+2y}{2x+3y}$	**A1**	AG – must come from correct working. The position of the negative must be clear.
		4	

Question	Answer	Marks	Guidance
7(b)	Equate $\frac{\mathrm{d}y}{\mathrm{d}x}$ to –2 and solve for x in terms of y or for y in terms of x	*M1	Must be using the given derivative.
	Obtain $x = -4y$ or $y = -\frac{x}{4}$	A1	Seen or implied by correct later work.
	Substitute *their* $x = -4y$ or *their* $y = -\frac{x}{4}$ in curve equation	DM1	Allow unsimplified.
	Obtain $y = \pm\frac{1}{\sqrt{7}}$ or $x = \pm\frac{4}{\sqrt{7}}$	A1	Or exact equivalent. Or $x = \frac{4}{\sqrt{7}}$ and $y = -\frac{1}{\sqrt{7}}$ or exact equivalent.
	Obtain both pairs of values	A1	Or $x = -\frac{4}{\sqrt{7}}$ and $y = \frac{1}{\sqrt{7}}$ or exact equivalent. A1 A0 for incorrect final pairing.
		5	

Question	Answer	Marks	Guidance
8(a)	Separate variables correctly	**B1**	$\int \frac{1}{4+9y^2}\mathrm{d}y = \int \mathrm{e}^{-(2x+1)}\mathrm{d}x$. Condone missing integral signs **or** dx and dy missing.
	Obtain term $-\frac{1}{2}\mathrm{e}^{-2x-1}$	**B1**	OE e.g. $-\frac{1}{2\mathrm{e}}\mathrm{e}^{-2x}$.
	Obtain term of the form $a\tan^{-1}\left(\frac{3y}{2}\right)$	**M1**	
	Obtain term $\frac{1}{6}\tan^{-1}\left(\frac{3y}{2}\right)$	**A1**	OE e.g. $\frac{1}{9}\times\frac{3}{2}\tan^{-1}\frac{3y}{2}$.
	Use $x = 1$, $y = 0$ to evaluate a constant or as limits in a solution containing or derived from terms of the form $a\tan^{-1}(by)$ and $c\mathrm{e}^{\pm(2x+1)}$	**M1**	If they rearrange before evaluating the constant, the constant must be of the correct form.
	Obtain correct answer in any form	**A1**	e.g. $\frac{1}{6}\tan^{-1}\frac{3y}{2} = \frac{1}{2}\mathrm{e}^{-3} - \frac{1}{2}\mathrm{e}^{-(2x+1)}$.
	Obtain final answer $y = \frac{2}{3}\tan\left(3\mathrm{e}^{-3} - 3\mathrm{e}^{-2x-1}\right)$	**A1**	OE Allow with $3\mathrm{e}^{-3} = 0.149....$
		7	
8(b)	State that y approaches $\frac{2}{3}\tan\left(3\mathrm{e}^{-3}\right)$	**B1 FT**	Or exact equivalent. The FT is on correct work on a solution containing e^{-2x-1}. Condone $y = \ldots$ Accept correct answer stated with minimal wording. 0.10032… is not exact so B0.
		1	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{1+2x}+\dfrac{B}{2-x}+\dfrac{C}{(2-x)^2}$	B1	Alternative form: $\dfrac{A}{1+2x}+\dfrac{Dx+E}{(2-x)^2}$
	Use a correct method for finding a coefficient	M1	e.g. $A(2-x)^2+B(1+2x)(2-x)+C(1+2x)$ $=2x^2+17x-17$ and compare coefficients or substitute for x. $A(2-x)^3+B(1+2x)(2-x)^2+C(1+2x)(2-x)$ $=2x^2+17x-17$ scores M0.
	Obtain one of $A=-4$, $B=-3$ and $C=5$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	Extra term in partial fractions, then B0 unless recover at end. Allow the marks for any constants found correctly. Missing terms in partial fractions, B0 but M1A1 is available for a correct method that obtains at least one correct constant (e.g. cover-up rule) Max 2/5. Ignore any substitution back into their original expression. If alternative form used: $A=-4$, $D=3$ and $E=-1$.
		5	

Question	Answer	Marks	Guidance
9(b)	Integrate and obtain terms $-2\ln(1+2x)+3\ln(2-x)+\dfrac{5}{2-x}$	B1FT B1FT B1FT	OE The FT is on correct use of *their* A, B and C; or on A, D and E. If using the A, D, E form then B1 for the A term, but no further marks until partial fractions are used to split the second term or they use integration by parts to obtain $\dfrac{Dx+E}{2-x}-\int\dfrac{D}{2-x}\mathrm{d}x$ for the 2nd B1 and 3rd B1 for correct completion. B0FT, B0FT, B0FT if they place *their* A, B, C with incorrect denominators.
	Substitute limits correctly in an integral with two terms (obtained correctly) of the form $a\ln(1+2x)+b\ln(2-x)+\dfrac{c}{2-x}$, where $abc\neq 0$	M1	Condone minor slips in substitution. Exact substitution required.
	Obtain answer $\dfrac{5}{2}-\ln 72$ after full and correct working	A1	AG – evidence of some correct work to combine or simplify logs is required e.g. allow from $-\ln 9+\ln\dfrac{1}{8}$ or $-\ln 2^3-\ln 3^2$.
		5	

Question	Answer	Marks	Guidance
10(a)	Use the product rule correctly to obtain $p(x+5)(3-2x)^n + q(3-2x)^{\frac{1}{2}}$	*M1	Allow with incorrect chain rule. BOD over sign errors unless an incorrect rule is quoted.
	Obtain correct derivative in any form	A1	e.g. $-(x+5)(3-2x)^{-\frac{1}{2}} + (3-2x)^{\frac{1}{2}}$.
	Equate derivative to zero and obtain a linear equation	DM1	Allow with surd factor e.g. $(3-2x)^{-\frac{1}{2}}\left(-(x+5)+(3-2x)\right)=0$.
	Obtain a correct linear equation.	A1	e.g. $-(x+5)+3-2x=0$.
	Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$.	A1	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. Accept with x, y stated separately. ISW
	Alternative Method for Question 10(a)		
	Obtain y^2 and differentiate	*M1	Ignore *their* left hand side i.e. *their* $\frac{\mathrm{d}}{\mathrm{d}x}y^2$.
	Obtain correct derivative in any form	A1	e.g. $-6x^2-34x-20$.
	Equate derivative to zero and solve for x	DM1	
	Obtain $-\frac{2}{3}$	A1	Ignore –5 if seen.
	Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$ only	A1	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. ISW
		5	

Question	Answer	Marks	Guidance
10(b)	Use the given substitution and reach $a\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\ \mathrm{d}u$	*M1	OE Need to see -2 or -½ used. Condone if du missing or the integral sign is missing. Allow M1A0 for complete substitution into $\int x\sqrt{3-2x}\,\mathrm{d}x$ to obtain first term of the line below.
	Obtain correct integral $-\frac{1}{2}\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\ \mathrm{d}u$	A1	OE e.g. $-\frac{1}{2}\left[\int\frac{3-u}{2}\sqrt{u}\,\mathrm{d}u+5\int\sqrt{u}\,\mathrm{d}u\right]$. Ignore limits at this stage. Condone if du missing.
	$x=-5$ and $\frac{3}{2}$	B1	SOI e.g. by u = 13 and 0. In any order and at any stage.
	Use correct limits the right way round in an integral of the form $a\left(\frac{26}{3}u^{\frac{3}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right)$	DM1	
	Obtain answer $\frac{169}{15}\sqrt{13}$ or $a=\frac{169}{15}$	A1	or exact equivalents.
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>11(a)</td><td>Carry out correct method for finding a vector equation for AB</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain [$\mathbf{r}$ =] $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$</td><td>A1</td><td>OE e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$.</td></tr>
<tr><td></td><td>Equate two pairs of components of general points on their AB and l
and evaluate λ or μ</td><td>M1</td><td>$\begin{pmatrix} 1+\lambda \\ 2-3\lambda \\ -2+3\lambda \end{pmatrix} = \begin{pmatrix} 1+2\mu \\ -1-3\mu \\ 3+4\mu \end{pmatrix}$.</td></tr>
<tr><td></td><td>Obtain correct answer for λ or μ, e.g. $\lambda = -1$, $\mu = -2$</td><td>A1</td><td>Correct value from two correct component equations.</td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and the lines fail to intersect
($\neq$ is sufficient justification e.g. $0 \neq -3$).</td><td>A1</td><td>Conclusion needs to follow correct values.
Hybrid versions are possible e.g. using j and k to get one parameter and then i to obtain the other.
or e.g. solving two pairs of simultaneous equations and showing that the results are not the same.
Alternatives:
<table>
<tr><td>A</td><td>λ</td><td>μ</td><td></td><td>B</td><td>λ</td><td>μ</td><td></td></tr>
<tr><td>ij</td><td>2</td><td>1</td><td>$4 \neq 7$</td><td>ij</td><td>1</td><td>1</td><td>$4 \neq 7$</td></tr>
<tr><td>ik</td><td>5</td><td>5/2</td><td>$-13 \neq$-17/2</td><td>ik</td><td>4</td><td>5/2</td><td>$-13 \neq$-17/2</td></tr>
<tr><td>jk</td><td>–1</td><td>–2</td><td>$0 \neq -3$</td><td>jk</td><td>–2</td><td>–2</td><td>$0 \neq -3$</td></tr>
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Question	Answer	Marks	Guidance
11(b)	Find $\overrightarrow{AP}$ for a general point P on l, e.g. $-3\mathbf{j} + 5\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$	**B1**	Or equivalent e.g. $\overrightarrow{PA} = -2\mu\mathbf{i} + (3\mu + 3)\mathbf{j} - (4\mu + 5)\mathbf{k}$.
	Calculate scalar product of *their* $\overrightarrow{AP}$ and a direction vector for l and equate the result to zero	**M1**	e.g. $4\mu + (9 + 9\mu) + (20 + 16\mu) = 0$. M0 if using $\overrightarrow{OP}$. M0 if using parallel line through A.
	Obtain $\mu = -1$	**A1**	
	Obtain answer $-\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	**A1**	Accept coordinates in place of position vector.
	Alternative Method for Question 11(b)		
	Find $\overrightarrow{AP}$ for a general point P on l, e.g. $-3\mathbf{j} + 5\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$	**B1**	Or equivalent e.g. $\overrightarrow{PA} = -2\mu\mathbf{i} + (3\mu + 3)\mathbf{j} - (4\mu + 5)\mathbf{k}$.
	Use Pythagoras and differentiate with respect to μ to obtain value of μ corresponding to minimum distance. (No need to prove it is a minimum)	**M1**	$\frac{d}{d\mu}\left(4\mu^2 + 9(\mu + 1)^2 + (4\mu + 5)^2\right) = 0$.
	Obtain $\mu = -1$	**A1**	
	Obtain answer $-\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	**A1**	Accept coordinates in place of position vector.
		4	